68th International Astronautical Congress (IAC), Adelaide, Australia, 25-29 September 2017.


IAC-17-A6.8.6

# Economic Valuation of Debris Removal

**Catherine Muller [a], Olga Rozanova [b*], Miguel Urdanoz [c]**

*a, b, c: Toulouse Business School.20 Boulevard Lascrosses, 31068 Toulouse.*
*SIRIUS Chair.*
*E-mails:*
*a: catherine.muller@tbs-education.fr.*
*b: olga.v.rozanova@gmail.com.*
*c: m.urdanoz@tbs-education.fr.*

**Abstract**

In the current paper we evaluate the lost expected value in the satellite-based service market that arises as a result of a catastrophic collision. We formalize the choice of the company regarding the number of the satellites it operates. The model allows expressing a lower bound for the surplus in the satellite-based service market via the observable variables (i.e. probability of satellite collision with debris and costs of satellite provision). We demonstrate that the destruction of $K$ satellites leads to loss in the expected surplus from service provision. This loss exceeds the costs needed to reconstruct the destroyed assets. Furthermore, we discuss the benefits of cleaning the space. In particular we show that debris removal leads to the reduction in the number of satellites chosen by the satellite operator.

**Key words:** debris removal; cleaning the space; choice of the number of satellites; surplus lower bound estimation.

## Nomenclature

$S$-satellite operator/owner.
$N$- number of satellites.
$p$- per satellite probability destruction caused by the collision with debris.
$T$- planning horizon.
$C$- satellite cost (construction plus launching).
$\pi$- per period profit of the satellite operator if at least one satellite is in order.
$r$-interest rate.
$\delta$-discount factor.
$\Pi$- perceived present discounted (to period zero) profit of satellite owner.
$E$- present discounted (to period zero) expenditures of satellite owner.
$CS$- per period consumer surplus in the satellite-based service market when at least one satellite is in order.
$W$- per period social surplus/ social welfare when at least one satellite is in order.
$W_t^{exp}[M]$- expected welfare/surplus in period $t$ given that $M$ satellites are in order at the moment $t-1$.
$K$-number of satellites destroyed because of the collision with debris.

## 1. Introduction

This paper contributes to the evaluation of the benefits of active space debris removal, or alternatively to the discussion of the costs of "doing nothing" (i.e. costs of not removing space debris). The literature usually defines the costs of "doing nothing" by the average expected costs of the assets, that were damaged in a catastrophic collision (see, for example, Levin and Carroll (2012)). However, besides the costs of the damaged assets, catastrophic collision entails some economic lost value (e.g., lost surpluses in the related markets, lost return from the alternative use of the resources spent on the reconstruction of the damaged asset etc.). To the best of our knowledge this economic value is not sufficiently studied in the literature.
In the current paper we evaluate the lost expected

value in the satellite-based service market that arises as a result of a catastrophic collision.

Our model is based on the idea that the number of satellites held by a company is a choice of the company itself. The company may decide to have some spare capacity (the examples of such companies include Iridium, Oneweb) or to operate a single satellite. The firm's own choice implies that this decision comes from some optimization problem. We formalize the choice of the company regarding the number of the satellites it operates. The solution to the optimization problem allows to express the company's profit as a function of the probability of collisions and the satellite costs, construction plus launching. This model allows expressing a lower bound for the surplus in the satellite-based service market via the known (or estimated in the literature) variables $p$ and $C$.[1]

We demonstrate that the destruction of $K \geq 1$ satellites leads to loss in the expected surplus from service provision. This loss weakly exceeds $C \cdot K$ (i.e. the costs needed to reconstruct the destroyed assets). In other words, the costs of "doing nothing" are much higher than the ones generally considered in the literature.

To the best of our knowledge the literature that applies **economic modeling** to the debris removal/ mitigation questions is very limited. The examples of papers that use economic toolbox to the debris removal/mitigation problem include Adilov et al. (2015), Macauley (2015). These papers, however, touch on different issues compared to the current paper. Namely, they discuss alternative remedies that encourage the firms to implement better debris mitigation techniques. In the current work we evaluate the lower bound of the loss in the welfare that arises as a result of **not** removing debris. Probably, the article that is the closest in spirit (in the satellite literature) to the present paper is the one by Odenwald et al. (2006), where the authors evaluate the loss in revenue that accompanies the damage in satellites caused by the 1859-calibre superstorm. The objective of this latter paper is similar to the one in the current work. However, the context and the modelling approaches are substantially different.

The structure of the rest of the paper is the following: Section 2 briefly presents the model. Section 3 calculates the loss in the expected surplus from the destruction of $K$ satellites. Section 4 discusses the benefit from cleaning the space. After that the concluding remarks follow.

## 2. Model

There is one satellite operator/owner, $S$, that provides some satellite-based services. $S$ has $N$ satellites. The provision of each satellite costs $C$. The planning horizon (that is the number of periods that the firm is going to function) is equal to $T$.

The timing is the following: At zero stage the company decides on the number of satellites that it is going to have. After that, in every subsequent period (during $T$ periods) it provides some services (if at least one satellite is in order) and it gets the payoff $\pi$. If, however, in some period $\tau \leq T$ there is no satellite that is in order, then the firm $S$ does not provide the service and in this case it gets zero payoff.

Potentially (in the model) the assets launched at period zero can be destroyed twice: at the first period (i.e. $\tau = 1$) and at period $\tau = t$. To simplify the calculations we assume that the satellite operator is myopic in the sense that it is aware about the possibility of satellites' collision with debris at $\tau = 1$, but it does not foresee it for the period $\tau = t$.

It is assumed here that the collision with debris is the only reason for satellite destruction. If $N$ satellites are launched at period zero, then the per period probability [calculated by $S$ at $\tau = 0$ moment of the game] that at least one satellite is functioning properly is equal to $(1 - p^N)$.[2] It is easy to see that the higher the $N$, the higher the $(1 - p^N)$.

At stage 0 the company $S$ decides on the number of satellites. On the one hand $S$ would like to have

[1] See, for example, Bai and Chen (2008), Xu and Xiong (2014) for the estimation of $p$. The examples of papers that estimate $C$ include Bearden (2001) and Wiedemann et al.(2003).

[2] The per period probability that at least one satellite is in order is equal to $(1 - p^N)$ under Assumption 2. If Assumption 2 is not satisfied, then this probability takes a more complicated form.

more satellites in order to raise the chances that at least one of them is in order (and, therefore, that $S$ gets $\pi$). On the other hand, $S$ prefer to decrease the number of satellites it launches since the provision of each additional satellite is costly (it costs $C$).

**Assumption 1:** $\frac{\partial p}{\partial N} = 0$.
Assumption 1 implies that the number of satellites launched by $S$ does not affect the probability of destruction of each satellite. The explanation for this assumption is the fact that $S$ may program the orbits of the satellites in such a way that they do not collide with each other. It should be noted, however, that in spite of the possibility to predetermine the trajectories of movements of the satellites, the launch of the additional satellite may affect the future probability of collision with debris. The reason for this is the following: the launch of the additional satellite weakly increases the number of objects in the neighborhood of any satellite $i$. The destruction of $i$ (because of the collision with debris) affects the number of debris in any neighborhood $\varepsilon$ of other satellites. The amount of debris in the neighborhood $\varepsilon$ may depend on the number of satellites launched. For the sake of simplicity in the current work we ignore this channel and stick to Assumption 1. It is worth mentioning that **qualitatively** our results are going to hold if to relax Assumption 1.

**Assumption 2:** The probability of satellite destruction (i.e. $p$) does not depend on the number of assets damaged due to the catastrophic collision with debris.

Assumption 2 is introduced for the sake of simplicity. In general it should not necessarily hold. The explanation for why Assumption 2 may not potentially be true is the following: destruction in some assets due to the collision with debris increases the number of debris in the space and, as a result, increases the probability of future collisions with debris of the satellites that are currently in order. Assumption 2 implies that the effect just described is not strong. More precisely, it suggests that the destruction of the satellite does not increase significantly the amount of debris in the space and, therefore, does not affect strongly the probability of future collisions with debris.[3]

## 3. Choice of the number of satellites

The discount factor is equal to $\delta = \frac{1}{1+r}$ (given that $r$ is the per period interest rate).
The expected discounted profit of firm $S$ is:[4]

$$\Pi = D(1-p^N)\pi - N \cdot C, \tag{1}$$

where $D = \delta(1+\delta+...+\delta^T) = \frac{\delta(1-\delta^{T+1})}{1-\delta}$.
The commonly used (expected) profit-maximization hypothesis is employed in the current analysis. In other words, it is assumed that satellite owner cares about its expected profit. Therefore, $N$ is chosen in a way that maximizes the present discounted profit of the firm $S$. Formally, the company $S$ solves the following optimization problem:

$$max_N \Pi = D(1-p^N)\pi - N \cdot C. \tag{2}$$

The above expected profit maximization program is equivalent to the following expenditure-minimization problem:

$$min_N E = D \cdot p^N \cdot \pi + N \cdot C. \tag{3}$$

The first summand in (3) (i.e. $D \cdot p^N \cdot \pi$) is the present discounted value of the expected profit that is forgone, if all the satellites turn out being out of order. The second component in (3) (i.e. $N \cdot C$) represent the costs incurred by $S$ on the provision/launch of $N$ satellites.
The first-order condition to (3) is:

$$\frac{\partial E}{\partial N} = D \cdot \pi \cdot p^N \ln p + C = 0. \tag{4}$$

The second-order condition to (3) is:

$$\frac{\partial^2 E}{\partial N^2} = D \cdot \pi \cdot p^N (\ln p)^2 > 0. \tag{5}$$

[3]Please, note that the main message of the paper continues being valid if to relax Assumption 2. However, relaxation of Assumption 2 complicates the mathematical expressions.

[4]Please, note that $\Pi$ takes the form given in (1) due to the fact that $S$ is myopic and does not foresees the possibility of satellites' destruction at $\tau = t$.

The second-order condition to (3) is globally satisfied. Therefore, indeed (4) allows getting the expenditure-minimizing value of $N$. Condition (4) defines $N^*$ that maximizes $\Pi$ (and minimizes $E$). Formally:

$$N^* = argmax_N(\Pi) = argmin_N(E). \quad (6)$$

To find $N^*$ the number of satellites (i.e. $N$) was treated as a continuous variable, while in fact it is discrete. It means that the actual number of satellites chosen by the company $S$ will be either higher or lower than $N^*$. In case $N^*$ turns out being integer (and non-negative), the actual number of satellites is going to coincide with $N^*$.
Let $\tilde{N}$ be the actual number of satellites provided/ launched by $S$.
If $\tilde{N} > N^*$, then due to (5) it can be concluded that $\frac{\partial E}{\partial N}|_{\tilde{N}>N^*} > 0$. In case $\tilde{N} < N^*$, the opposite is true (i.e. $\frac{\partial E}{\partial N}|_{\tilde{N}<N^*} < 0$).
Using (4) the following system represents the different cases in a laconic way:

$$\begin{cases} D\cdot\pi\cdot p^{\tilde{N}}\ln p + C = 0, \ if\ \tilde{N} = N^* \\ D\cdot\pi\cdot p^{\tilde{N}}\ln p + C > 0, \ if\ \tilde{N} > N^* \\ D\cdot\pi\cdot p^{\tilde{N}}\ln p + C < 0, \ if\ \tilde{N} < N^*. \end{cases} \quad (7)$$

System (7) may be rewritten in the following way:

$$\begin{cases} \pi = \dfrac{C}{D\cdot p^{\tilde{N}}\cdot|\ln p|}, \ if\ \tilde{N} = N^* \\ \pi < \dfrac{C}{D\cdot p^{\tilde{N}}\cdot|\ln p|}, \ if\ \tilde{N} > N^* \\ \pi > \dfrac{C}{D\cdot p^{\tilde{N}}\cdot|\ln p|}, \ if\ \tilde{N} < N^*. \end{cases} \quad (8)$$

System (8) allows expressing the value of $\pi$ (which is usually unobservable) in terms of the variables ($C$, $p$, $r$, $\tilde{N}$ ) the values of which are usually known (because they are either observable or estimated in the literature).
What is crucial is that (8) provides the exact value of or the lower bound for $\pi$ for the case $\tilde{N} \leq N^*$. As for the case where $\tilde{N} > N^*$, the expression in (8) defines the upper bound for $\pi$.
In the current analysis we are interested in the lower bound (or the exact value) for $\pi$. So, we can use the results in (8) for $\tilde{N} \leq N^*$.

**Lower bound for $\pi$, when $\tilde{N} > N^*$.**
To find the lower bound for $\pi$ for the case $\tilde{N} > N^*$ let us use the following reasoning:
The fact that the firm $S$ chooses $\tilde{N}$ satellites implies that it is better-off (or at least it is not worse-off) by making this choice than by choosing $(\tilde{N}-1)$ satellites. Firm $S$ is better-off in the sense that its (perceived) profit (i.e. $\Pi$) is higher (or equivalently, its costs (i.e. $E$) are lower) when it chooses $N = \tilde{N}$, than under $N = \tilde{N}-1$.
Formally this condition is written in the following way:

$$\Pi|_{N=\tilde{N}} \geq \Pi|_{N=\tilde{N}-1}. \quad (9)$$

Using (1) and (9) we get:

$$\pi \geq \frac{C}{D\cdot p^{N-1}(1-p)}|_{N=\tilde{N}}. \quad (10)$$

**Lemma 1.** $A = \frac{1-p}{p\cdot|\ln p|} > 1$.

**Proof:** The proof of Lemma 1 follows directly from the following observations:

1. $\frac{\partial A}{\partial p} = \frac{\ln p + (1-p)}{(p\ln p)^2} < 0$. The inequality follows from the fact that $\ln p < -1$ (since $p < 1$).
2. $lim_{p\to 1}A = lim_{p\to 1}\frac{1-p}{-p\cdot\ln p} = 1$. To see that $lim_{p\to 1}A = 1$ it is enough to make a substitution $p = 1+x$ and apply the "beautiful" limit $lim_{x\to 0}\frac{\ln(1+x)}{x} = 1$.

From the above statements it follows that when $p$ approaches 1, $A$ tends to 1 as well. Since $A$ is decreasing in $p$, the reduction in $p$ leads to rise in $A$. So, it may be concluded that for $p \ll 1$ $A > 1$. Q.E.D.

The expression (10) gives the lower bound for $\pi$ for any $\tilde{N}$ (i.e. for both $\tilde{N} \leq N^*$ and for $\tilde{N} > N^*$). So, for the case $\tilde{N} \leq N^*$ both expressions (i.e. the one given by (10) and the one presented in (8)) are the lower bounds for $\pi$. However, Lemma 1 implies that the value of the bound defined in (8) exceeds the one in (10). Therefore, (8) gives a more accurate value of the lower bound for $\pi$ for the case $\tilde{N} \leq N^*$ (more precisely, when $\tilde{N} = N^*$ the expression in (8) presents the exact value of $\pi$ instead of its lower bound).
Proposition 1 summarizes the above discussion.

**Proposition 1.**

1. *For $\tilde{N} = N^*$ the value of $\pi$ is $\frac{C}{D \cdot p^{\tilde{N}}|\ln p|}$ (i.e. $\pi = \frac{C}{D \cdot p^{\tilde{N}}|\ln p|}$).*
2. *For $\tilde{N} < N^*$ the value of the lower bound for $\pi$ is equal to $\frac{C}{D \cdot p^{\tilde{N}}|\ln p|}$ (i.e. $\pi > \frac{C}{D \cdot p^{\tilde{N}}|\ln p|}$).*
3. *For $\tilde{N} > N^*$ the value of the lower bound for $\pi$ is equal to $\frac{C}{D \cdot p^{N-1}(1-p)}|_{N=\tilde{N}}$ (i.e. $\pi \geq \frac{C}{D \cdot p^{N-1}(1-p)}|_{N=\tilde{N}}$).*

### 3.1 *Social welfare*

The total welfare is the sum of the surpluses of all the agents participating in the market. In the satellite-based service market besides the satellite operator there are consumers of the satellite-based services. Using the notations of the paper, the per period welfare in case at least one satellite is in order is:

$$W = \pi + CS. \tag{11}$$

To estimate the consumer surplus the information about the demand for satellite-based services is needed. This information is often hard to collect/access.

The revealed preferences argument tells that the agents decide to participate in the market as long as their benefits/surpluses from participation exceed the once received if they quit the market. Assuming that the consumers are rational,this allows to conclude that since they participate in the market their (expected) surplus is (weakly) greater than zero (i.e. $CS^{exp} = Pr \cdot CS + (1 - \mathrm{Pr}) \cdot 0 \geq 0 \Rightarrow CS \geq 0$).[5]

The above discussion implies that the social welfare (social surplus), which is equal to the sum of the surpluses of all the agents, is (weakly) greater than the profit of satellite owner. More precisely the per period welfare in case when at least one satellite is in order (weakly) exceeds $\pi$. Therefore, the lower bound for $\pi$ may be interpreted as the lower bound for $W$. Corollary 1 summarizes this discussion.

**Corollary 1.** *The lower bound for $\pi$ also defines the lower bound for $W$.*

1. $W \geq \pi \geq \frac{C}{D \cdot p^{\tilde{N}}|\ln p|}$ *for $\tilde{N} \leq N^*$.*
2. $W \geq \pi > \frac{C}{D \cdot p^{\tilde{N}-1}(1-p)}$ *for $\tilde{N} > N^*$.*

[5] $Pr$ is the probability that at least one satellite is in order.

## 4. Loss in expected surplus

Assume that all $N$ satellites launched at moment zero are still in order at period $t-1$, then the expected discounted welfare at period/moment $t$ takes the form:

$$W_t^{exp}[N] = (1 + \delta + \delta^2 + ... + \delta^{T-t})(1 - p^N)W.$$

Taking into account that $1 + \delta + \delta^2 + ... + \delta^{T-t} = \frac{1-\delta^{T-t+1}}{1-\delta}$, $W_t^{exp}[N]$ can be rewritten as:

$$W_t^{exp}[N] = \frac{1 - \delta^{T-t+1}}{1 - \delta}(1 - p^N)W. \tag{12}$$

Now assume that till period $t$ $K$ satellites were destroyed because of the collision with debris [more precisely, in our framework it means that these $K$ satellites were destroyed at period $\tau = 1$]. Then the expected discounted welfare/surplus at period $t$ is:

$$W_t^{exp}[N-K] = \frac{1 - \delta^{T-t+1}}{1 - \delta}(1 - p^{N-K})W. \tag{13}$$

The difference in the levels of the expected welfare between the scenarios, where all $N$ satellites are in order at period $t-1$ and where just $(N-K)$ satellites function well at $t-1$ (i.e. $\triangle W_t^{exp} = W_t^{exp}[N] - W_t^{exp}[N-K]$) is:

$$\triangle W_t^{exp} = \frac{1 - \delta^{T-t+1}}{1 - \delta} p^{N-K}(1 - p^K)W. \tag{14}$$

Social surplus (welfare) is usually non-observable and it is difficult to estimate it. So, $W$ in (14) is not known. However, Corollary 1 gives us the lower bound of $W$. This allows receiving the lower bound for the loss in expected surplus at period $t$ (i.e. $\triangle W_t^{exp}$) that takes place because of the destruction of $K$ satellites. Proposition 2 presents this lower bound for $\triangle W_t^{exp}$.

**Proposition 2.**

1. $\triangle W_t^{exp} \geq \frac{1-\delta^{T-t+1}}{\delta(1-\delta^{T+1})} \cdot \frac{C(1-p^K)}{p^K|\ln p|}$ *for* $\tilde{N} \leq N^*$.

2. $\triangle W_t^{exp} > \frac{1-\delta^{T-t+1}}{\delta(1-\delta^{T+1})} \cdot \frac{C(1-p^K)}{p^{K-1}(1-p)}$ *for* $\tilde{N} > N^*$.

**Proof:** The proof of Proposition 2 follows directly from (14) and Corollary 1. Q.E.D.

It is worth noting that the minimum bound for $\triangle W_t^{exp}$ given in Proposition 2 does not depend on the level of $N$.

Another important observation is that for a very large $T$ (formally, for $T \to \infty$) $\frac{1-\delta^{T-t+1}}{\delta(1-\delta^{T+1})} \to \frac{1}{\delta}$. Corollary 2 gives the lower bound of $\triangle W_t^{exp}$ for the case where $T$ is infinitely large.

**Corollary 2.** *For* $T \to \infty$*:*

1. $\triangle W_t^{exp} \geq \frac{1}{\delta} \cdot \frac{C(1-p^K)}{p^K|\ln p|}$ *for* $\tilde{N} \leq N^*$.

2. $\triangle W_t^{exp} > \frac{1}{\delta} \cdot \frac{C(1-p^K)}{p^{K-1}(1-p)}$ *for* $\tilde{N} > N^*$.

**Proof:** The proof of Corollary 2 follows directly from Proposition 2 and the fact that $\frac{1-\delta^{T-t+1}}{\delta(1-\delta^{T+1})} \to \frac{1}{\delta}$ when $T \to \infty$. Q.E.D.

Once again for the case where $T \to \infty$ our approach allows expressing the lower bound for the loss in the welfare from the destruction of $K$ satellite as a function of observable variables (namely, via $p$, $C$, $\delta$ (or $r$)).

The intuition for why this forgone expected surplus presents is straightforward:

- Every period the surplus $W$ is gained if at least one satellite is in order.

- The probability that at least one satellite functions properly is equal to one minus the probability that all the satellites are destroyed. So, if $N$ satellites are in order in the current period, then the probability that at least one satellite will function well in the next period is equal to $(1-p^N)$. If $(N-K)$ satellites are in order now, then the probability that at least one satellite will be in order in the next period is equal to $(1-p^{N-K})$.

- Thus, the destruction of $K$ satellites reduces the probability of gaining the surplus/welfare $W$ in the future from $(1-p^N)$ to $(1-p^{N-K})$.

As it was already mentioned in the introduction, the literature usually considers the costs of the destroyed assets, when it calculates the losses from not cleaning the space/not removing the debris. In a basic model described here these losses are equal to $C \cdot K$.

The analysis made indicates, however, that there are additional costs, that are equal to the forgone expected surplus that is not gained due to the destruction of the satellites. Using the notations employed in the paper this forgone expected surplus is equal to $\triangle W_t^{exp}$.

Proposition 3 states that $\triangle W_t^{exp}$ is always higher (or equal) to $C \cdot K$. In other words, the size of the forgone expected surplus is significant and should not be neglected when calculating the losses from not cleaning the space. Moreover, Proposition 3 emphasizes that the difference between the lower bound of $\triangle W_t^{exp}$ and $C \cdot K$ increases with rise in $K$. Saying it differently, the higher is the number of the destroyed satellites, the more crucial becomes the loss in the expected surplus/welfare.

**Proposition 3.** *Let* $T \to \infty$*, then:*

1. *For any* $K$ $\triangle W_t^{exp} \geq C \cdot K$.

2. *Rise in* $K$ *increases the gap between the lower bound of* $\triangle W_t^{exp}$ *and* $C \cdot K$.

**Proof:** The two steps are used for the proof of Proposition 3. Firstly, we show that at $K = 1$ the lower bound of $\triangle W_t^{exp}$ is higher or equal to $C \cdot K$. Secondly, we demonstrate that the derivative with respect to $K$ of the difference between the lower bound of $\triangle W_t^{exp}$ and $C \cdot K$ is positive.

Let $K = 1$, then using Corollary 2 we may conclude that:

- $\triangle W_t^{exp} \geq \frac{1}{\delta} \cdot \frac{C(1-p)}{p|\ln p|}$ for $\tilde{N} \leq N^*$.

- $\triangle W_t^{exp} > \frac{1}{\delta} \cdot C$ for $\tilde{N} > N^*$.

Using Lemma 1 and the fact that $\delta = \frac{1}{1+r} \leq 1$ we may conclude that at $K = 1$ the lower bound for $\triangle W_t^{exp}$ is higher (or equal) to $C$ (i.e. $C \cdot K$ at $K = 1$). Formally, for $K = 1$:

- $\triangle W_t^{exp} \geq \frac{1}{\delta} \cdot \frac{C(1-p)}{p|\ln p|} > C$ for $\tilde{N} \leq N^*$.
- $\triangle W_t^{exp} > \frac{1}{\delta} \cdot C \geq C$ for $\tilde{N} > N^*$.

Now let us consider the difference between the lower bound of $\triangle W_t^{exp}$ and $C \cdot K$. The two cases are examined (i.e.$\tilde{N} \leq N^*$ and $\tilde{N} > N^*$).

**Case 1:** $\tilde{N} \leq N^*$
From Corollary 2 $\triangle W_t^{exp} \geq \frac{1}{\delta} \cdot \frac{C(1-p^K)}{p^K|\ln p|}$.

$$\frac{\partial(\frac{1}{\delta} \cdot \frac{C(1-p^K)}{p^K|\ln p|} - C \cdot K)}{\partial K} = \frac{C(1-\delta \cdot p^K)}{\delta \cdot p^K} > 0.$$

**Case 2:** $\tilde{N} > N^*$
From Corollary 2 $\triangle W_t^{exp} > \frac{1}{\delta} \cdot \frac{C(1-p^K)}{p^{K-1}(1-p)}$.

$$\frac{\partial(\frac{1}{\delta} \cdot \frac{C(1-p^K)}{p^{K-1}(1-p)} - C \cdot K)}{\partial K} = \frac{C(|\ln p| - \delta(1-p)p^{K-1})}{\delta(1-p)p^{K-1}} > 0.$$

The signs of the above derivatives imply that the second statement in Proposition 3 is true.
If to add to this the fact that at $K = 1$ the lower bound of $\triangle W_t^{exp}$ is higher or equal to $C \cdot K$, then it becomes obvious that the first statement in Proposition 3 is also true. Q.E.D.

## 5. Benefits from cleaning the space

So far we have discussed the losses from **not** cleaning the space. More precisely, we defined and calculated the lost expected
welfare/surplus in the satellite-based service market that arises as a result of the destruction of $K$ satellites.
Here we are going to consider the benefits from cleaning the space. Two types of benefits are discussed. They are:

- Decrease in the probability of loosing the surplus $W$ (or alternatively, increase in the probability of gaining the surplus $W$).
- Decrease in the number of satellites that the company $S$ chooses to operate.

### 5.1 *Effect on the probability of gaining the surplus at the satellite-based service market*

Cleaning the space leads to the reduction in the per period probability of the satellite destruction. This probability reduces from $p$ to $\hat{p} < p$. As a result, the probability that all $N$ satellites are destroyed reduces from $p^N$ to $\hat{p}^N$. Thus, the likelihood of gaining $W$ (i.e. the likelihood that at least one satellite is in order) increases from $(1 - p^N)$ to $(1 - \hat{p}^N)$.

### 5.2 *Decrease in the number of satellites the company chooses to operate*

Here we prove that cleaning the space leads to lower $N$ chosen by satellite operator than it would have been the case without debris removal. The intuition for this outcome is the following: cleaning the space reduces the probability of catastrophic collisions, and, therefore, increases the likelihood of gaining $\pi$ and $W$ for any given $N$. It means that satellite operator needs lower $N$ in order to ensure the same probability of getting $\pi$ and $W$.
Proposition 4 formally states and proves this idea.

**Proposition 4.** $\frac{\partial N^*}{\partial p} > 0$.

**Proof:** $N^*$ is found from (4), that is:

$$\frac{\partial E}{\partial N} = D \cdot \pi \cdot p^N \ln p + C = 0.$$

Using the implicit function theorem we get:

$$\frac{\partial N^*}{\partial p} = -\frac{\frac{\partial^2 E}{\partial N \partial p}}{\frac{\partial^2 E}{\partial N^2}}$$

.
From (5) $\frac{\partial^2 E}{\partial N^2} > 0$.
$\frac{\partial^2 E}{\partial N \partial p} = D \cdot \pi \cdot p^{N-1}(N \cdot \ln p + 1) < 0$.

Therefore, $\frac{\partial N^*}{\partial p} > 0$. Q.E.D.

The result in Proposition 4 suggests that cleaning the space reduces the number of satellites that the satellite operator chooses to launch. This allows to save the resources that would have otherwise been spent.

## 6. Concluding remarks

In the paper we demonstrated that the destruction of the satellites (because of the catastrophic collision) entails the loss in the expected surplus in the satellite-based service market. This is a new kind of loss from not cleaning the space that was not previously discussed in the literature on debris removal.

Furthermore, we modeled the satellite operator's choice of the number of satellites that it chooses to have. The modeling approach allowed us to express the lower bound of the loss in the expected welfare (that arises because of the destruction of $K$ satellites) in terms of the per satellite probability of collision and the (construction plus launching) costs of the satellite (i.e. in terms of the variables, whose estimates are known from the previous literature).

We prove that the loss in the expected surplus is (weakly) larger than the costs of the damaged assets (i.e. the loss from not cleaning the space that is usually emphasized in the literature). In other words, the decrease in the expected welfare is significant and should not be neglected when calculating the losses from not cleaning the space.

Also we show that cleaning the space leads to reduction in the number of satellites the the satellite operator chooses to have. Saying it differently, cleaning the space allows saving some resources that would have otherwise been spent. The same idea can be expressed in terms of the losses from **not** cleaning the space: not cleaning the space entails more satellites that the satellite operator chooses to have (compared to the scenario under which the space is cleaned).

## References

[1] N. Adilov, P. J. Alexander, and B. M. Cunningham. An economic analysis of earth orbit pollution. *Environmental and Resource Economics*, 60(1):81–98, 2015.

[2] X.-z. Bai and L. Chen. Research on calculational method of collision probability between space objects [j]. *Journal of Astronautics*, 4:065, 2008.

[3] D. A. Bearden. Small-satellite costs. *Crosslink*, 2(1):32–44, 2001.

[4] E. M. Levin and J. A. Carroll. The cost of future collisions in leo. *White Paper, February*, 28, 2012.

[5] M. K. Macauley. The economics of space debris: Estimating the costs and benefits of debris mitigation. *Acta Astronautica*, 115:160–164, 2015.

[6] S. Odenwald, J. Green, and W. Taylor. Forecasting the impact of an 1859-calibre superstorm on satellite resources. *Advances in Space Research*, 38(2):280–297, 2006.

[7] C. Wiedemann, M. Oswald, J. Bendisch, H. Sdunnus, and P. Voersmann. Cost of space debris mitigation measures. In *AIAA Space 2003 Conference & Exposition*, page 6212, 2003.

[8] X.-L. Xu and Y.-Q. Xiong. A method for calculating probability of collision between space objects. *Research in Astronomy and Astrophysics*, 14(5):601, 2014.